Exploring players' adaptation to non-Euclidean video game environments

Video Games for Research

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Abstract

In the research community, the studies around the concept of non-Euclidean video games usually include either popular mechanics, such as portals, or Virtual Reality (VR) sets, which is an easy way to simulate an environment far from the real world. This study explores players' adaptation to non-Euclidean video game environments and measures their overall performance based on different metrics of the game and their background. We developed a video game with both Euclidean and non-Euclidean levels, defined two hypotheses and conducted a survey that included 22 participants. The results show that participants' adaptation in non-Euclidean environments depends on the presence of reference objects and individual factors. Participants could adapt well with constant visible references. Without these, adaptation relies more on individual traits, specifically game experience, whether they are action game players and their navigation strategies.

The online version of our game can be found here. The GitHub repository of the project can be found here.

Keywords- non-Euclidean, video game, navigation

1 Introduction

In this section, we introduce some preliminary knowledge of non-Euclidean geometry. Then, we provide some study results that are relevant to the present study and define our main research question, state the two hypotheses, our motivation, and some video game examples that incorporate non-Euclidean environments or mechanics.

1.1 Background

In mathematics, there are different types of geometries depending on the rules (or *axioms*) that apply to each case. The most famous one is Euclidean Geometry, the geometry of the plane (zero curvature), which respects the five Postulates of Euclid [14]. These postulates describe the construction of this geometry. For instance, Euclid states that a straight line can be drawn from any point to any point, all right angles are equal, and π is a constant number that describes the area of any circle divided by the square root of its radius. However, there are cases where some of the Postulates of Euclid are violated. The most common violation is observed in the fifth postulate (or *parallel postulate*), which states that are less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles" [13]. In case of such a violation, which implies changes in the axioms, this type of geometry is called non-Euclidean Geometry. The two traditional non-Euclidean geometries are the Hyperbolic and Elliptic Geometry [12]. Figure 1 shows some main differences between axioms of different geometries. Figure 2 depicts how a triangle is constructed in a Hyperbolic, Euclidean and Elliptic Space respectively.

Aside from the strictly mathematical definition, the term "non-Euclidean" is often used by gamers and developers to describe a game space that is different from the real world [7]. The game industry already



Figure 1: Axioms of Euclidean, Elliptic and Hyperbolic Geometry. Image from [3].

has a wide variety of non-Euclidean Geometry-inspired video games to offer. It is worth mentioning that there is a distinction between non-Euclidean video games (or video games in a non-Euclidean environment) and video games that include non-Euclidean mechanics (usually 3D video games). Popular non-Euclidean video games include Hyperbolica¹, HyperRogue² and Antichamber³. Some non-Euclidean mechanics that are currently trending in the gaming world are infinite loops, forced perspective, gravity manipulation and shifting between 2D and 3D spaces. Popular video games that incorporate non-Euclidean mechanics include Portal⁴, Superliminal⁵ and Manifold Garden⁶.



Figure 2: Triangles in a Hyperbolic, Euclidean and Elliptic Space. Image from [1].

1.2 Related Work

In the research community, many studies around non-Euclidean video game environments have been conducted, with Hyperbolic spaces being the most famous concept. Nevertheless, most of the studies we could find use the non-Euclidean mechanic of loops (or "wormholes"), and researchers are particularly interested in testing non-Euclidean video games with Virtual Reality (VR) sets.

In [4], the authors examined the participants' ability to choose the shortest path to reach a target in a maze. There were two kinds of mazes selected; a physically realizable, which is closer to the 3D world and easy to mentally represent, and a physically impossible, which includes "wormholes" and is closer to a non-Euclidean reality. The main task was to identify what kind of information a participant uses to choose a path when they are at a junction. The authors observed that the participants' trajectories became progressively closer to the shortest path, and they behaved differently in the two environments. Overall, participants' strategy selection depended on the complexity of the environment, for example, the number of corridors or twists and turns in the maze. In [5], the goal of the research was to test human navigation in a hyperbolic space. The hypothesis was that people can navigate in Hyperbolic space relatively as easily as in a Euclidean space. For the testing, the authors designed and used a simple VR game with Euclidean and Hyperbolic levels to compare players' navigation in the different spaces. The results showed that, after confusion and mistakes, the players were able to adapt to the hyperbolic space. In some cases, the differences between the spaces were not clear to the players, and the navigation was even done more intuitively than in Euclidean space. Therefore, the initial hypothesis was proven correct. In [2], the goal was to examine how players perceive and navigate in non-Euclidean environments by using an immersive VR game that creates the impression of a non-Euclidean environment. The game was tested with four

¹https://store.steampowered.com/app/1256230/Hyperbolica/

²https://store.steampowered.com/app/342610/HyperRogue/

³https://store.steampowered.com/app/219890/Antichamber/

⁴https://store.steampowered.com/app/400/Portal/

⁵https://store.steampowered.com/app/1049410/Superliminal/

⁶https://store.steampowered.com/app/473950/Manifold_Garden/

participants and the results showed that players' orientation and overall performance were not negatively affected by the non-Euclidean elements. In [10], the authors made the hypothesis that people operate as in a Euclidean space, even when put into a non-Euclidean space and curvature violations are seen. Two experiments were conducted using both Euclidean and non-Euclidean (hyperbolic and spherical) spaces. The results showed that this hypothesis is correct. In a later study [11], the same authors examined the types of Euclidean representations people may form while learning virtual wormhole mazes to determine if the nature of spatial representations that people develop is based on Euclidean geometry. Participants explored Euclidean or non-Euclidean tunnel mazes and drew maps of the landmark layout on a 2D canvas. The results showed that people developed different strategies, either based on Euclidean (preserving distance information) or non-Euclidean features (preserving turning angles). Therefore, strategies vary among individuals. Warren et al. 9 compared three hypotheses about how people use spatial knowledge to navigate in virtual environments: they use a cognitive map with Euclidean structure and consistent coordinates, a network of paths, or a labeled graph that stores metric information. In the experiments, participants navigated in a non-Euclidean environment with "wormholes" that could teleport them. The results showed that "the knowledge of navigation space is best characterized by a labeled graph, in which local metric information is approximate, geometrically inconsistent, and not embedded in a common coordinate system". Finally, in a study two years later [8], Warren argues that human behavior in video games is more likely to be based on a cognitive graph representation rather than a Euclidean cognitive map. After testing navigation in a non-Euclidean environment, it was shown that performance relies on the environment's features but, at the same time, the results support the cognitive graph theory, which seemed to apply in both Euclidean and non-Euclidean environments.

1.3 Research Objective

As mentioned in Section 1.2, the studies around the concept of non-Euclidean geometries in video games focus mostly on non-Euclidean mechanics rather than non-Euclidean environments. This deficit of information motivated the present work, since it would be of great interest to put players in a non-Euclidean video game environment and document their navigation strategies, difficulties and general behavior in the game. Essentially, the players will be exposed to a non-intuitive, far from the real-world environment rather than non-Euclidean mechanics.

The present study aims to explore how players adapt to a non-Euclidean environment incorporated in a video game. Two hypotheses have been defined for this purpose;

Hypothesis 1: Non-experienced players struggle to navigate

Hypothesis 2: Players who play action video games have greater visual attention in the game area and therefore perform better

The first hypothesis is based on intuition. We consider experienced players to usually be exposed to many different video game environments and therefore develop more and diverse strategies and perceptual skills depending on the game and the game environment. On the other hand, non-experienced players are considered to be more susceptible to illusions and tend to follow more intuitive navigation strategies stemming from the real world.

The second hypothesis is inspired by the study results in Pöhlmann et al. [6]. The authors state that action video game players show "enhanced visual perceptual functions compared to their non-video game playing peers". In our case, we aim to see if action video games have any influence on players' perception of the game environment and if there is a further connection to players' performance.

To answer these hypotheses, we developed a puzzle-like video game with both Euclidean and non-Euclidean levels. We also created a survey to gather background and performance information of participants, as well as insights, limitations and suggestions.

2 Research Methods

Our research was carried out using two methods: a survey consisting of a pre-game and post-game part, and a game to be played. The pre-game survey was used to gather information on the player to perform our analysis on. This information concerned age, time spent playing video games, which genre of games the participant mostly played, and their familiarity with non-Euclidean games. The post-game survey was used to gather experiences from the game itself. This part was split up for each level, asking specific questions on the participant's experience with each level such as what they found difficult, what they found enjoyable, and what they found confusing. The survey can be found in Appendix A.

To find out how well players with these different backgrounds can navigate hyperbolic space, we devised three levels that each have a Euclidean and non-Euclidean version. Initially, the player is presented with the Euclidean level, so we can measure their baseline performance in the regular three dimensions, and also allow the player to familiarize themselves with the layout, which will be important as we will describe soon. The levels consist of collecting the coins placed in the level itself, with the level completing as soon as all coins are collected. We measure the time taken to collect all coins, starting when the player first collects a coin.

Before each pair of levels is presented to the player, a message is shown on-screen describing the objective of each level, namely collecting coins. We also emphasize that it is important for the player to take note of the locations of the coins in the Euclidean version of the level, as they will be in the same location in the non-Euclidean level. They are allowed to use any way of remembering the coin placements, such as noting down directions, tile colors or, if applicable, tile coordinates.

The player is allowed to move forward and backward through the pair of levels. However, once the player moves on to the next pair of levels, we prohibit access to previous levels.

The player's controls are WASD for moving around, the mouse for looking around, Q/left click for opening up the mini-map, providing a top-down perspective of the level, and the ENTER/BACKSPACE keys to cycle forwards and backwards through the levels. The mini-map allows the player to have a better sense of direction, especially in the non-Euclidean levels, where geometries quickly move beyond the horizon due to the perspective projection required. The player is allowed to move forwards to the next level if they feel stuck, or that they can't finish the stage, and they are also allowed to move backwards to the previous level to take better note of the coin locations in the Euclidean level. The player can walk past the boundaries of the map without falling off (they will continue walking as if the surface is still there), but exploring too far beyond the edge can render the player lost if they lose sight of the starting tiles. We don't expect players to wander out of bounds, however, as there is nothing to be found here.

2.1 Level 1: Straight Line

The first level is purposefully easy to get the player familiar with the game and the concepts of (non-) Euclidean space. The levels contain two coins, the first placed one tile ahead of the player and one coin placed at the edge of the level, six tiles away. The purpose of this level is to see if participants can move in a straight line in a non-Euclidean world as well as they can in a Euclidean world.



Figure 3: Forward half of the layout of Euclidean (left) and non-Euclidean (right) level 1. The player must walk in a straight line to collect all coins. Elements not to scale.

2.2 Level 2: Exploring

The second level is a step up in difficulty, as the map now contains ten coins. The first three coins are in a straight line from the origin, an the remaining seven are evenly spaced around the perimeter of the map. Each coin has a unique color, which will be useful for the post-game survey when inquiring about which coins were the most difficult to find. We also have four mathematical land markers, namely a +, -, \times and π symbol, placed around the map to provide players with some orientation. For the Euclidean



Figure 4: Mini-maps of Euclidean (left) and non-Euclidean (right) level 1

map, these are not very useful as all coins are visible from any location, but can help orient the player in the non-Euclidean map, where the total space covered by the tiles is exponentially greater.

In this level, the tiles are labeled by the shortest path to get to a tile, with first the left-right axis and then the up-down axis. For instance, a tile whose shortest path from the origin is three up, two right is labeled RRUUU. For the non-Euclidean world, the same is done; however, tiles are present here that are not in the Euclidean world. For example, moving up, then right moves you onto a different tile than moving right, then up. It is therefore crucial that a player takes note of the tile where each coin is present, and uses this information to find the coins in the non-Euclidean world.

The goal is to collect all coins, but we predict not all players will be able to do so. This level requires careful note-taking of the coins, the way non-Euclidean tiles are labeled, and a good sense of direction.



Figure 5: Layout of Euclidean (left) and non-Euclidean (right) level 2. The player must take note of the coordinates of the tiles containing coins in the Euclidean level, as they are the same for the non-Euclidean level. The violation of Euclid's fifth postulate (*parallel postulate*) is shown clearly. Four mathematical land markers are placed around the map to provide some orientation for the player. Elements not to scale.



Figure 6: Mini-maps of Euclidean (left) and non-Euclidean (right) level 2

2.3 Level 3: Maze

The third level consists of a maze where the player is tasked to navigate the maze and find the single coin placed outside of its boundaries. The purpose of this level is to see if participants can navigate the non-Euclidean maze more quickly when already having been shown the layout of the maze in the Euclidean world. The maze itself is very basic, with only two dead-ends which can be avoided easily.



Figure 7: Layout of Euclidean (left) and non-Euclidean (right) level 3. The player must remember the path through the Euclidean maze to solve the non-Euclidean maze. Elements not to scale.



Figure 8: Mini-maps of Euclidean (left) and non-Euclidean (right) level 3

3 Results

In total, 22 players participated in our study. According to the pre-survey, their ages are mainly in the range of 22-26 years old, with one player younger than 21 years and one player older than 31 years. Players have varied gaming experience, with 36.36% playing less than 1 hour per week, 18.18% playing 1-5 hours per week, and 45.45% playing more than 5 hours per week. 10 out of 22 players, consider themselves as action game players, which is 45.45%. Additionally, except for one Mobile Phone player and one Console player, the remaining 20 players usually play games on PCs and therefore have a basic knowledge of moving in computer games.

3.1 Level 1: Straight Line

For Level 1, which allows players to walk straight, the results are shown in Figure 9.



Figure 9: Time(s) spent in Euclidean vs non-Euclidean worlds in Level 1

Due to the fact that one player continued to get lost in the non-Euclidean environment for several attempts and was unable to complete the walking in a straight line task, there were only 21 valid data. Based on Figure 9, it can be seen that 5 players in the valid data also got lost in the non-Euclidean environment and retried. Only 7 players took less time to walk a straight line in the non-Euclidean environment than in the Euclidean environment. However, according to the post-survey, only 22.73% of the players found it somewhat difficult or difficult.

3.2 Level 2: Exploring

For level 2, the non-Euclidean environment has only map markers (letters) and scattered reference objects that are not always visible, the result is shown in Figure 10. Since one player's time record was lost, there were a total of 21 valid data. Only 8 players managed to find all the coins in the valid data, 3 of them had a second try. In the post-survey, almost all players found level 2 the most difficult. In order to find out what makes some players more adapted to non-Euclidean environments than others, we analyzed the players' backgrounds and the navigation strategies used and found 3 important influencing factors: gaming experience, action game player or not, and navigation strategy.



Figure 10: Number of coins collected and the time(s) taken in the non-Euclidean environment in Level 2

3.2.1 Hypothesis 1: Game experience

As shown in Figure 11(a), overall the more time players spend playing the game per week, the better they perform in the non-Euclidean environment in Level 2, which means that they can collect more coins. Players who play the game for less than 1 hour per week generally only collect less than 6 coins, except for 2 outlier data points. These two players play the game for less than 1 hour per week and still collect all the coins in level 2. After removing these 2 outliers, an Analysis of variance (ANOVA) test was performed on the data, showing the p-value is 0.013 < 0.05. This indicates that the players' weekly playing time shows a strong correlation with the coin number they collected. Thus it can be concluded that our hypothesis 1 stands, those with gaming experience are more adaptable in non-Euclidean environments and those non-experienced players struggle to navigate.



(a) Playtime per week vs performance in non-Euclidean level 2

(b) Action game players vs performance in non-Euclidean level 2

Figure 11: Box charts for Hypothesis 1 and Hypothesis 2

3.2.2 Hypothesis 2: Action game player

As shown in Figure 11(b), in general, when players consider themselves to be action game players, they are more adapted to the non-Euclidean environment and collect more coins in level 2. An Independent samples t-test was taken on the data and the p-value is 0.050, which suggests that being an action game player or not indeed affects performance in the non-Euclidean environment. The conclusion of our hypothesis 2 holds.

3.2.3 Navigation strategies

In the post-survey, we investigated some open questions to study players' navigation strategies in the non-Euclidean environment. Based on the results of the questionnaire, we summarised the players' strategies into four categories: no strategy, walk around or only remember where to turn, write down the letter or first go straight then explore, use letter hints and find the rule/pattern of letters. We analyze the relationship between players' navigation strategies and their adaptation performance in the non-Euclidean environment, the results of which are shown in Figure 12. Players who were able to use letter hints and find the rule all collected all coins. Players who noted the letters or walked straight first collected more than 7 coins. whereas the other two categories of players who had no strategy or walked randomly collected almost less than 6 coins, except for one outlier. Evidently, the navigation strategy used by the players is also critical in the non-Euclidean environment.



Figure 12: Number of coins collected and strategies used in the non-Euclidean environment in Level 2

3.3 Level 3: Maze

In the maze level, which has visible reference objects along the way and a mini-map, the results are shown in Figure 13. No players got lost or retried at this level. 16 out of 22 players found the coin in the non-Euclidean world faster or almost equal to the Euclidean world when the location of the coin was known.



Figure 13: A scatter plot comparing the time(s) spent in Euclidean and non-Euclidean worlds in Level 3

4 Discussion and Limitations

4.1 Discussion

Based on the above results, we found that when players first encounter the non-Euclidean world, even if they just need to complete simple tasks, like walking in a straight line, they would still get lost. However, after initial familiarisation with the environment, most players do not consider themselves to have difficulty adapting to the non-Euclidean environment.

The difficulty rises significantly when the player is in a situation where there are no consistently visible reference objects and the players need to explore the map in order to complete the task. In this case, if the players have game experience or are action game players, they will adapt to the non-Euclidean environment better than others, as we hypothesized. If players use objective navigation strategies and have an overall sense of the space, they would also be more adapted to non-Euclidean environments.

Furthermore, navigation is not a big problem in non-Euclidean environments when there are visual reference objects along the way. Players would navigate as if they were operating in the Euclidean environment, which is consistent with the findings of the previous study [10].

4.2 Limitations

While summarising and analyzing the data, we noted several limitations in our study. Since we did not videotape every player's gameplay, some of the phenomena found in the subsequent analyses were difficult to count. For example, in the case of mini-map use, some players mentioned the use of mini-maps in the open questions, while some players did not use it. This may have implications for navigation strategies as well as player performance in non-Euclidean environments. But unfortunately, we don't have a record of it.

In addition, we found 3 outlier data in the analysis of Hypothesis 1 and navigation strategies in level 2, but it is difficult to confirm whether they were different from other players in other performances based on the available data.

Finally, in our study, we observed that some players would be nervous and unwilling to attempt level 2 more than twice due to us watching them because they thought it would take too long, which is a pity.

5 Conclusion and Future Work

In order to extend the existing research on players' behavior in non-Euclidean geometry-inspired video games and turn the interest towards non-Euclidean environments with simple mechanics, the current study was executed, which consisted of a custom-made video game and a survey. With the help of 22 participants, different factors related to gameplay were measured and two hypotheses were to be evaluated.

We conclude that participants' adaptation in non-Euclidean environments depends on the surroundings. When there are always visible reference objects, participants behave as if they were in the Euclidean world and adapt well. However, when reference objects were scattered and not always visible, participants' ability to adapt depended more on the individual, and there were three important influencing factors: game experience, whether participants were action game players or not, and participants' navigation strategies. When participants have more gaming experience and are action game players, their adaptation ability in non-Euclidean environments is significantly better than others. Thus both hypotheses we proposed were true. And objective navigation strategies would also help participants adapt to the non-Euclidean environment.

Finally, using the current findings, a continuation of this research could investigate the impact of non-Euclidean video game environments on players' spatial reasoning, cognitive skills, or eye-hand coordination. It would be of great interest to discover whether this exploration of spatial behaviors in non-Euclidean environments is associated with players' attention or immersion and consequently decisionmaking and performance in the video game.

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A Appendix: Survey



Part 1: Player's background

What is your age?

O 21 or younger

○ 22-26

○ 27-30

31 or older

How much time do you spend on video games in a week?

less than one hour
more than one hour, less than five hours
more than five hours

What type of video games do you like to play?

Action games
First-person shooter (FPS) games
Role-playing games (RPG)
Adventure games
Simulation games
Strategy games
] Sports games
Other, please specify:

Do you consider yourself an action game player?

) Yes			
) No			

Do you consider yourself a first-person shooter (fps) game player?

○ Yes			
O No			

Where do you usually play video games?

D PC
Game console
Mobile phone
Other, please specify:

Do you know what a non-Euclidean game is?

○ Yes			
O No			

Have you ever played a non-Euclidean game before? If yes, which one?

O Yes			
O No			

Part 2: In the game

LEVEL 1

How long did it take to collect the coins in the **Euclidean** world?

How long did it take to collect the coins in the **non-Euclidean** world?

LEVEL 2

How many coins did you collect in the **non-Euclidean** world? How long did it take?

LEVEL 3

How long did it take to collect the coins in the **Euclidean** world?

How long did it take to collect the coins in the **non-Euclidean** world?

Part 3: After the game

LEVEL 1

How difficult did you find it to walk in a straight line in the **non-Euclidean** world?

O Very difficult	
O Somewhat difficult	
O Neither easy nor difficult	
O Somewhat easy	
O Very easy	

LEVEL 2

Both **Euclidean** and **non-Euclidean** worlds in level 2 included some mathematical symbols. Did you use them while playing the game? If yes, how?

Do you think the mathematical symbols made the directions clearer or more confusing for you?

Describe the strategy you used to find coins in the **non-Euclidean** world.

LEVEL 3

Which features did you enjoy?

Which features did you find confusing or frustrating?

Overall, which of the three levels did you find most difficult? Why?

B Appendix: Results

B.1 Players' background



B.2 Post-game results



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